Decay-lepton angular distributions in $e^+e^- \to t\bar{t}$ to $\mathcal{O}(\alpha_s)$ in the soft-gluon approximation

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Abstract

Order-α_s QCD corrections in the soft-gluon approximation to angular distributions of decay charged leptons in the process $e^+e^- \to t\bar{t}$, followed by semileptonic decay of t or $\bar{t}$, are obtained in the e^+e^- centre-of-mass frame. As compared to distributions in the top rest frame, these have the advantage that they would allow direct comparison with experiment without the need to reconstruct the top rest frame or a spin quantization axis. Analytic expressions for the distribution in the charged-lepton polar angle, and triple distribution in the polar angle of t and polar and azimuthal angles of the lepton are obtained. Numerical values for the polar-angle distributions of charged leptons are discussed for $\sqrt{s} = 400$ GeV and 800 GeV.

1   Introduction

The discovery [1] of a heavy top quark, with a mass of about 174 GeV which is close to the electroweak symmetry breaking scale, raises the interesting possibility that the study of its properties will provide hints to the mechanism of symmetry breaking. While most of the gross properties of the top quark will be investigated at the Tevatron and LHC, more accurate determination of its couplings will have to await the construction of a linear e^+e^- collider. The prospects of the construction of such a linear collider, which will provide detailed information also on the $W^\pm$, Z and Higgs, are under

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intense discussion currently, and it is very important at the present time to focus on the details of the physics issues ([2] and references therein).

In this context, top polarization is of great interest. There has been a lot of work on production of polarized top quarks in the standard model (SM) in hadron [3] collisions, and e^+e^- collisions in the continuum [4], as well as at the threshold [5]. A comparison of the theoretical predictions for single-top polarization as well as $t\bar{t}$ spin correlations with experiment can provide a verification of SM couplings and QCD corrections, or give clues to possible new physics beyond SM in the couplings of the top quark [6-15] (see [16] for a review of CP violation in top physics).

Undoubtedly, the study of the top polarization is possible because of its large mass, which ensures that the top decays fast enough for spin information not to be lost due to hadronization [17]. Thus, kinematic distributions of top decay products can be analysed to obtain polarization information. It would thus be expedient to make predictions directly for kinematic distributions rather than for the polarization of the top quarks, as is usually done. Such an approach makes the issue of the choice of spin basis for the top quark (see the discussion on the advantage of the "off-diagonal" basis in [18, 19], for example) superfluous. Moreover, if the study is restricted to energy and polar angle distributions of top decay products, it even obviates the need for accurate determination of the energy or momentum direction of the top quark [7].

In this paper we shall be concerned with the laboratory-frame angular distribution of secondary leptons arising from the decay of the top quarks in $e^+e^- \to t\bar{t}$ in the context of QCD corrections to order α_s. QCD corrections to top polarization in $e^+e^- \to t\bar{t}$ have been calculated earlier by many groups [20-26]. QCD corrections to decay-lepton angular distributions in the top rest frame have been discussed in [20]. QCD corrections to the lepton energy distributions have been treated in the top rest frame in [15] and in the laboratory (lab.) frame in [10]. This paper provides, for the first time, angular distribution in the e^+e^- centre-of-mass (c.m.) frame. As a first approach, this work is restricted, for simplicity, to the soft-gluon approximation (SGA). SGA has been found to give a satisfactory description of top polarization in single-top production [26], and it is hoped that it will suffice to give a reasonable quantitative description.

The study of the lab.-frame angular distribution of secondary leptons, besides admitting direct experimental obervation, has another advantage. It

has been found [12, 13] that the angular distribution is not altered, to first-order approximation, by modifications of the tbW decay vertex, provided the b-quark mass is neglected. Thus, our result would hold to a high degree of accuracy even when $\mathcal{O}(\alpha_s)$ soft-gluon QCD corrections to top decay are included, since these can be represented by the same form factors [28] considered in [12, 13]. We do not, therefore, need to calculate these explicitly. It is sufficient to include $\mathcal{O}(\alpha_s)$ corrections to the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices. This, of course, assumes that QCD corrections of the nonfactorizable type [29], where a virtual gluon is exchanged gluon between t $(\bar{t})$ and $\bar{b}$ (b) from $\bar{t}$ (t) decay, can be neglected. We have assumed that these are negligible.

The procedure adopted here is as follows. We make use of effective $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices derived in earlier works in the soft-gluon approximation, using an appropriate cut-off on the soft-gluon energy. In principle, these effective vertices are obtained by suitably cancelling the infra-red divergences in the virtual-gluon contribution to the differential cross section for $e^+e^- \to t\bar{t}$ against the real soft-gluon contribution to the differential cross section for $e^+e^- \to t\bar{t}g$. For practical purposes, restricting to SGA, it is sufficient to modify the tree-level $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices suitably to produce the desired result. Thus, assuming $\mathcal{O}(\alpha_s)$ effective SGA vertices, we have obtained helicity amplitudes for $e^+e^- \to t\bar{t}$, and hence spin-density matrices for production. This implies an assumption that these effective vertices provide, in SGA, a correct approximate description of the off-diagonal density matrix elements as well as the diagonal ones entering the differential cross sections. Justification for this would need explicit calculation of hard-gluon effects, and is beyond the scope of this work.

We have considered three possibilities, corresponding to the electron beam being unpolarized $(P = 0)$, fully left-handed polarized $(P = -1)$, and fully right-handed polarized $(P = +1)$. Since we give explicit analytical expressions, suitable modification to more realistic polarizations would be straightforward.

Our main result may be summarized as follows. By and large the distribution in the polar angle θ_l of the secondary lepton w.r.t. the e^- beam direction is unchanged in shape on inclusion of QCD corrections in SGA. The θ_l distribution for $\sqrt{s} = 400$ GeV is very accurately described by overall multiplication by a K factor ($K \equiv 1 + \kappa > 1$), except for extreme values of θ_l, and that too for the case of $P = +1$. For $\sqrt{s} = 800 GeV$, κ continues to be slowly varying function of θ_l. This has the important consequence that

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earlier results on the sensitivity of lepton angular distributions or asymmetries to anomalous top couplings, obtained for $\sqrt{s}$ values around 400 GeV without QCD corrections being taken into account, would go through by a simple modification by a factor of $1/\sqrt{K}$.

2 Expressions

We first obtain expressions for helicity amplitudes for

$$e^-(p_{e^-}) + e^+(p_{e^+}) \rightarrow t(p_t) + \bar{t}(p_{\bar{t}}) \tag{1}$$

going through virtual γ and Z in the e^+e^- c.m. frame, including QCD corrections in SGA. The starting point is the QCD-modified $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices obtained earlier (see, for example, [26, 27]). We can write them [26] in the limit of vanishing electron mass as

$$\Gamma^\gamma_\mu = e\left[c_v^\gamma \gamma_\mu + c_M^\gamma \frac{(p_t - p_{\bar{t}})_\mu}{2m_t}\right], \tag{2}$$

$$\Gamma^Z_\mu = e\left[c_v^Z \gamma_\mu + c_a^Z \gamma_\mu \gamma_5 + c_M^Z \frac{(p_t - p_{\bar{t}})_\mu}{2m_t}\right], \tag{3}$$

where

$$c_v^\gamma = \frac{2}{3}(1 + A), \tag{4}$$

$$c_v^Z = \frac{1}{\sin\theta_W \cos\theta_W}\left(\frac{1}{4} - \frac{2}{3}\sin^2\theta_W\right)(1 + A), \tag{5}$$

$$c_a^\gamma = 0, \tag{6}$$

$$c_a^Z = \frac{1}{\sin\theta_W \cos\theta_W}\left(-\frac{1}{4}\right)(1 + A + 2B), \tag{7}$$

$$c_M^\gamma = \frac{2}{3}B, \tag{8}$$

$$c_M^Z = \frac{1}{\sin\theta_W \cos\theta_W}\left(\frac{1}{4} - \frac{2}{3}\sin^2\theta_W\right)B. \tag{9}$$

The form factors A and B are given to order α_s in SGA by

$$\mathrm{Re}A = \hat{\alpha}_s \left[\left(\frac{1+\beta^2}{\beta} \log \frac{1+\beta}{1-\beta} - 2 \right) \log \frac{4\omega_{\max}^2}{m_t^2} - 4 \right.$$
$$+ \frac{2+3\beta^2}{\beta} \log \frac{1+\beta}{1-\beta} + \frac{1+\beta^2}{\beta} \left\{ \log \frac{1-\beta}{1+\beta} \left(3 \log \frac{2\beta}{1+\beta} \right. \right.$$
$$\left. \left. \left. + \log \frac{2\beta}{1-\beta} \right) + 4\mathrm{Li}_2 \left(\frac{1-\beta}{1+\beta} \right) + \frac{1}{3}\pi^2 \right\} \right], \tag{10}$$

$$\mathrm{Re}B = \hat{\alpha}_s \frac{1-\beta^2}{\beta} \log \frac{1+\beta}{1-\beta}, \tag{11}$$

$$\mathrm{Im}B = -\hat{\alpha}_s \pi \frac{1-\beta^2}{\beta}, \tag{12}$$

where $\hat{\alpha}_s = \alpha_s/(3\pi)$, $\beta = \sqrt{1 - 4m_t^2/s}$, and Li_2 is the Spence function. $\mathrm{Re}A$ in eq. (10) contains the effective form factor for a cut-off $\omega_{\max}$ on the gluon energy after the infrared singularities have been cancelled between the virtual- and soft-gluon contributions in the on-shell renormalization scheme. Only the real part of the form factor A has been given, because the contribution of the imaginary part is proportional to the Z width, and hence negligibly small [23, 26]. The imaginary part of B, however, contributes to azimuthal distributions.

The vertices in eqs. (2) and (3) can be used to obtain helicity amplitudes for $e^+e^- \to t\bar{t}$, including the contribution of s-channel γ and Z exchanges. The result is, in a notation where the subscripts of M denote the signs of the helicities of e^-, e^+, t and $\bar{t}$, in that order,

$$M_{+-\pm\pm} = \pm \frac{4e^2}{s} \sin\theta_t \frac{1}{\gamma} \left[\left(c_v^\gamma + r_R c_v^Z \right) - \beta^2 \gamma^2 \left(c_M^\gamma + r_R c_M^Z \right) \right], \tag{13}$$

$$M_{-+\pm\pm} = \pm \frac{4e^2}{s} \sin\theta_t \frac{1}{\gamma} \left[\left(c_v^\gamma + r_L c_v^Z \right) - \beta^2 \gamma^2 \left(c_M^\gamma + r_L c_M^Z \right) \right], \tag{14}$$

$$M_{+-\pm\mp} = \frac{4e^2}{s} (1 \pm \cos\theta_t) \left[\pm \left(c_v^\gamma + r_R c_v^Z \right) + \beta \left(c_a^\gamma + r_R c_a^Z \right) \right], \tag{15}$$

$$M_{-+\pm\mp} = \frac{4e^2}{s} (1 \mp \cos\theta_t) \left[\mp \left(c_v^\gamma + r_L c_v^Z \right) - \beta \left(c_a^\gamma + r_L c_a^Z \right) \right], \tag{16}$$

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where θ_t is the angle the top-quark momentum makes with the e^- momentum, $\gamma = 1/\sqrt{1-\beta^2}$, and $r_{L,R}$ are related to the left- and right-handed $Ze\bar{e}$ couplings, and are given by

$$r_L = \left(\frac{s}{s - m_Z^2}\right) \frac{1}{\sin\theta_W \cos\theta_W}, \tag{17}$$

$$r_R = -\left(\frac{s}{s - m_Z^2}\right) \tan\theta_W. \tag{18}$$

Since we are interested in lepton distributions arising from top decay, we also evaluate the helicity amplitudes for $t \to bl^+\nu_l$ (or $\bar{t} \to \bar{b}l^-\overline{\nu}_l$), which will be combined with the production amplitudes in the narrow-width approximation for t ($\bar{t}$) and W^+ (W^-). In principle, QCD corrections should be included also in the decay process. However, in SGA, these could be written in terms of effective form factors [28]. As found earlier [12, 13], in the linear approximation, these form factors do not affect the charged-lepton angular distribution. Hence we need not calculate these form factors.

The decay helicity amplitudes in the t rest frame can be found in [13], and we do not repeat them here. We will simply make use of those results.

The final result for the angular distribution in the lab. frame can be written as

$$
\begin{aligned}
\frac{d^3\sigma}{d\cos\theta_t d\cos\theta_l d\phi_l} = {}& \frac{3\alpha^2\beta m_t^2}{8s^2} B_l \frac{1}{(1 - \beta\cos\theta_{tl})^3} \\
\times {}& \Big[\mathcal{A}(1 - \beta\cos\theta_{tl}) + \mathcal{B}(cos\theta_{tl} - \beta) \\
& + \mathcal{C}(1 - \beta^2)\sin\theta_t \sin\theta_l (\cos\theta_t \cos\phi_l - \sin\theta_t \cot\theta_l) \\
& + \mathcal{D}(1 - \beta^2)\sin\theta_t \sin\theta_l \sin\phi_l \Big],
\end{aligned}
\tag{19}
$$

where θ_t and θ_l are polar angles of respectively of the t and l^+ momenta with respect to the e^- beam direction chosen as the z axis, and ϕ_l is the azimuthal angle of the l^+ momentum relative to an axis chosen in the $t\bar{t}$ production plane. B_l is the leptonic branching ratio of the top. θ_{tl} is the angle between the t and l^+ directions, given by

$$\cos\theta_{tl} = \cos\theta_t \cos\theta_l + \sin\theta_t \sin\theta_l \cos\phi_l, \tag{20}$$

and the coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ are given by

$$\mathcal{A} = A_0 + A_1 \cos\theta_t + A_2 \cos^2\theta_t, \tag{21}$$
$$\mathcal{B} = B_0 + B_1 \cos\theta_t + B_2 \cos^2\theta_t, \tag{22}$$
$$\mathcal{C} = C_0 + C_1 \cos\theta_t, \tag{23}$$
$$\mathcal{D} = D_0 + D_1 \cos\theta_t, \tag{24}$$

with

$$
\begin{aligned}
A_0 &= 2\left\{(2-\beta^2)\left[2c_v^{\gamma 2} + 2(r_L+r_R)c_v^\gamma c_v^Z + (r_L^2+r_R^2)c_v^{Z2}\right]\right.\\
&\quad +\beta^2(r_L^2+r_R^2)c_a^{Z2} - 2\beta^2\left[2c_v^\gamma c_M^\gamma + (r_L+r_R)(c_v^\gamma c_M^Z + c_v^Z c_M^\gamma)\right.\\
&\quad \left.\left. +(r_L^2+r_R^2)c_v^Z c_M^Z\right]\right\}(1-P_e P_{\bar{e}})\\
&\quad +2\left\{(2-\beta^2)\left[2(r_L-r_R)c_v^\gamma c_v^Z + (r_L^2-r_R^2)c_v^{Z2}\right] + \beta^2(r_L^2-r_R^2)c_a^{Z2}\right.\\
&\quad \left. -2\beta^2\left[(r_L-r_R)(c_v^\gamma c_M^Z + c_v^Z c_M^\gamma) + (r_L^2-r_R^2)c_v^Z c_M^Z\right]\right\}(P_{\bar{e}}-P_e),\\
A_1 &= -8\beta c_a^Z\left\{\left[(r_L-r_R)c_v^\gamma + (r_L^2-r_R^2)c_v^Z\right](1-P_e P_{\bar{e}})\right.\\
&\quad \left. + \left[(r_L+r_R)c_v^\gamma + (r_L^2+r_R^2)c_v^Z\right](P_{\bar{e}}-P_e)\right\},\\
A_2 &= 2\beta^2\left\{\left[2c_v^{\gamma 2} + 4c_v^\gamma c_M^\gamma + 2(r_L+r_R)(c_v^\gamma c_v^Z + c_v^\gamma c_M^Z + c_v^Z c_M^\gamma)\right.\right.\\
&\quad \left. +(r_L^2+r_R^2)\left(c_v^{Z2} + c_a^{Z2} + 2c_v^Z c_M^Z\right)\right](1-P_e P_{\bar{e}})\\
&\quad + \left[2(r_L-r_R)(c_v^\gamma c_v^Z + c_v^\gamma c_M^Z + c_v^Z c_M^\gamma)\right.\\
&\quad \left.\left. +(r_L^2-r_R^2)\left(c_v^{Z2} + c_a^{Z2} + 2c_v^Z c_M^Z\right)\right](P_{\bar{e}}-P_e)\right\},\\
B_0 &= 4\beta\left\{\left(c_v^\gamma + r_L c_v^Z\right)r_L c_a^Z(1-P_e)(1+P_{\bar{e}})\right.\\
&\quad \left. + \left(c_v^\gamma + r_R c_v^Z\right)r_R c_a^Z(1+P_e)(1-P_{\bar{e}})\right\},\\
B_1 &= -4\left\{\left[(c_v^\gamma + r_L c_v^Z)^2 + \beta^2 r_L^2 c_a^{Z2}\right](1-P_e)(1+P_{\bar{e}})\right.\\
&\quad \left. - \left[(c_v^\gamma + r_R c_v^Z)^2 + \beta^2 r_R^2 c_a^{Z2}\right](1+P_e)(1-P_{\bar{e}})\right\},\\
B_2 &= 4\beta\left\{\left(c_v^\gamma + r_L c_v^Z\right)r_L c_a^Z(1-P_e)(1+P_{\bar{e}})\right.\\
&\quad \left. + \left(c_v^\gamma + r_R c_v^Z\right)r_R c_a^Z(1+P_e)(1-P_{\bar{e}})\right\},\\
C_0 &= 4\left\{\left[(c_v^\gamma + r_L c_v^Z)^2 - \beta^2\gamma^2\left(c_v^\gamma + r_L c_v^Z\right)\left(c_M^\gamma + r_L c_M^Z\right)\right](1-P_e)(1+P_{\bar{e}})\right.\\
&\quad \left. - \left[(c_v^\gamma + r_R c_v^Z)^2 - \beta^2\gamma^2\left(c_v^\gamma + r_R c_v^Z\right)\left(c_M^\gamma + r_R c_M^Z\right)\right](1+P_e)(1-P_{\bar{e}})\right\},
\end{aligned}
$$

$$C_1 = -4\beta\left\{\left[\left(c_v^\gamma + r_L c_v^Z\right) - \beta^2\gamma^2\left(c_M^\gamma + r_L c_M^Z\right)\right]r_L c_a^Z(1 - P_e)(1 + P_{\bar{e}})\right.$$
$$\left. + \left[\left(c_v^\gamma + r_R c_v^Z\right) - \beta^2\gamma^2\left(c_M^\gamma + r_R c_M^Z\right)\right]r_R c_a^Z(1 + P_e)(1 - P_{\bar{e}})\right\},$$
$$D_0 = 0,$$
$$D_1 = 0.$$

Integrating over ϕ_l and θ_{tl} we get the θ_l distribution:

$$\frac{d\sigma}{d\cos\theta_l} = \frac{3\pi\alpha^2}{32s}\beta B_l\left\{\left(4A_0 + \frac{4}{3}A_2\right) + \left[-2A_1\left(\frac{1-\beta^2}{\beta^2}\log\frac{1+\beta}{1-\beta} - \frac{2}{\beta}\right)\right.\right.$$
$$+2B_1\frac{1-\beta^2}{\beta^2}\left(\frac{1}{\beta}\log\frac{1+\beta}{1-\beta} - 2\right)$$
$$\left.+2C_0\frac{1-\beta^2}{\beta^2}\left(\frac{1-\beta^2}{\beta}\log\frac{1+\beta}{1-\beta} - 2\right)\right]\cos\theta_l$$
$$+\left[2A_2\left(\frac{1-\beta^2}{\beta^3}\log\frac{1+\beta}{1-\beta} - \frac{2}{3\beta^2}\left(3 - 2\beta^2\right)\right)\right.$$
$$+\frac{1-\beta^2}{\beta^3}\left\{B_2\left(\frac{\beta^2-3}{\beta}\log\frac{1+\beta}{1-\beta} + 6\right)\right.$$
$$\left.\left.-C_1\left(\frac{3(1-\beta^2)}{\beta}\log\frac{1+\beta}{1-\beta} - 2(3 - 2\beta^2)\right)\right\}\right]$$
$$\times(1 - 3\cos^2\theta_l)\right\}. \tag{25}$$

3   Numerical Results and Discussion

After having obtained analytic expressions for angular distributions, we now examine the numerical values of the QCD corrections. We will discuss only the θ_l distributions of (25), leaving a discussion of the triple distributions given in (19) for a future publication.

We use the parameters $\alpha = 1/128$, $\alpha_s(m_Z^2) = 0.118$, $m_Z = 91.187$ GeV, $m_W = 80.41$ GeV, $m_t = 175$ GeV and $\sin^2\theta_W = 0.2315$. We consider leptonic decays into one specific channel (electrons or muons or tau leptons), corresponding to a branching ratio of $1/9$. We have used, following [26], a gluon energy cut-off of $\omega_{\max} = (\sqrt{s} - 2m_t)/5$. While qualitative results
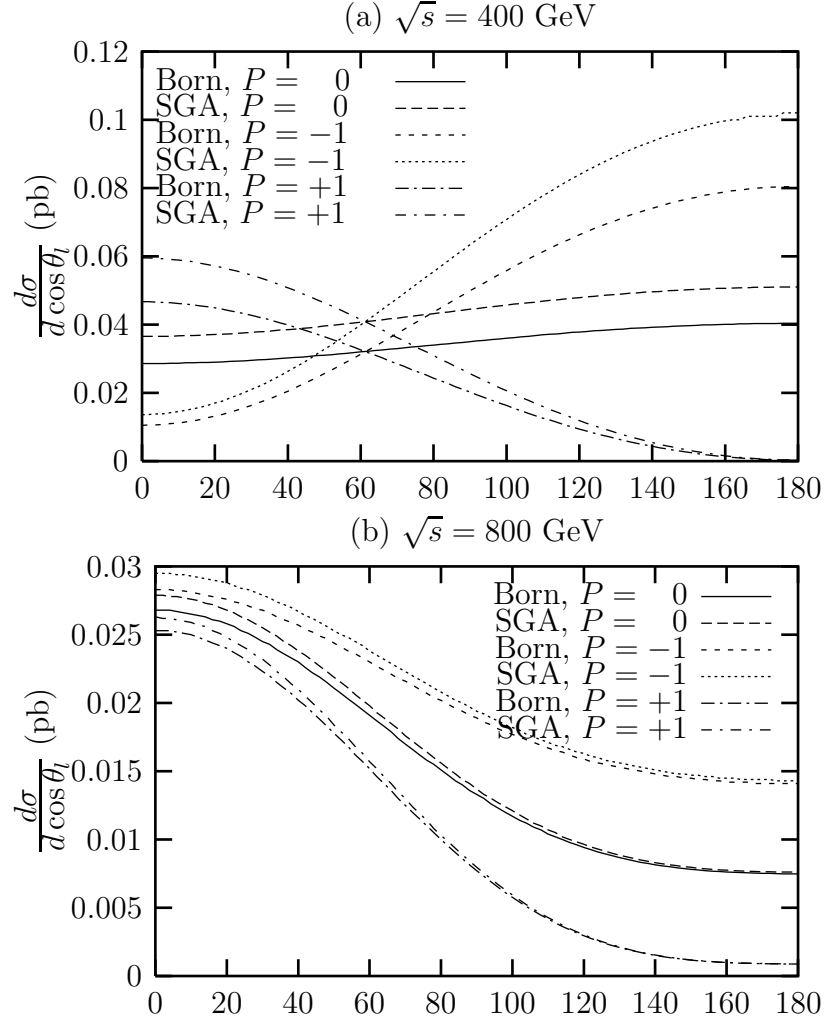
Figure 1: The distribution in θ_l with and without QCD corrections for (a) $\sqrt{s} = 400$ GeV and (b) $\sqrt{s} = 800$ GeV plotted against θ_l, for e^- beam polarizations $P = 0, -1, +1$ in each case.

would be insensitive, exact quantitative results would of course depend on the choice of cut-off.

In Fig. 1 we show the single differential cross section $\frac{d\sigma}{d\cos\theta_l}$ in picobarns with and without QCD corrections, for two values of $\sqrt{s}$, viz., (a) 400 GeV and (b) 800 GeV, and for e^- beam polarizations $P = 0, -1, +1$. It can be seen that the distribution with QCD corrections follows, in general, the shape of the lowest order distribution.

In Fig. 2 is displayed the fractional deviation of the QCD-corrected distribution from the lowest order distribution:

$$\kappa(\theta_l) = \left(\frac{d\sigma_{Born}}{d\cos\theta_l}\right)^{-1}\left(\frac{d\sigma_{SGA}}{d\cos\theta_l} - \frac{d\sigma_{Born}}{d\cos\theta_l}\right). \tag{26}$$

It can be seen that $\kappa(\theta_l)$ is independent of θ_l to a fair degree of accuracy for $\sqrt{s} = 400$ GeV.

In Fig. 3 we show the fractional QCD contributions $(F_{\text{SGA}} - F_{\text{Born}})/F_{\text{Born}}$ where $F(\theta_l)$, is the normalized distribution:

$$F(\theta_l) = \frac{1}{\sigma}\left(\frac{d\sigma}{d\cos\theta_l}\right). \tag{27}$$

It can be seen that the fractional change in the normalized distributions for $\sqrt{s} = 400$ GeV is at most of the order of 1 or 2% (except in the case of $P = +1$, for $\theta_l \geq 160°$). For the other values of $\sqrt{s}$, it is even smaller. This implies that QCD corrected angular distribution is well approximated, at the per cent level, by a constant rescaling by a K factor.

To conclude, we have obtained in this paper analytic expressions for angular distributions of leptons from top decay in $e^+e^- \to t\bar{t}$, in the e^+e^- c.m. frame, including QCD corrections to order α_s in the soft-gluon approximation. The distributions are in a form which can be compared directly with experiment. In particular, the single differential θ_l distribution needs neither the reconstruction of the top momentum direction nor the top rest frame. The triple differential distribution does need the top direction to be reconstructed for the definition of the angles. However, in either case the results do not depend on the choice of spin quantization axis.

We find that the θ_l distributions are well described by rescaling the zeroth order distributions by a factor K which for $\sqrt{s} = 400$ GeV is roughly independent of θ_l, except for extreme values of θ_l, for the case of right-handed
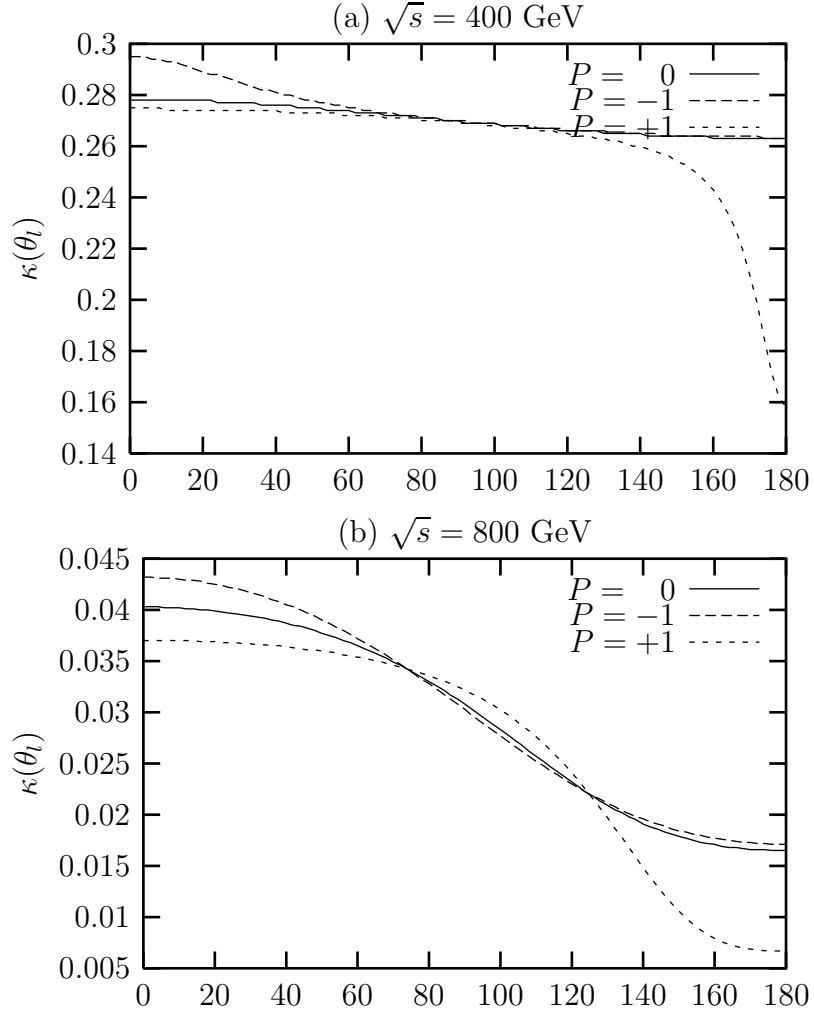
Figure 2: The fractional QCD contribution $\kappa(\theta_l)$ defined in the text for (a) $\sqrt{s} = 400$ GeV and (b) $\sqrt{s} = 800$ GeV plotted as a function of θ_l, for $P = 0, -1, +1$.
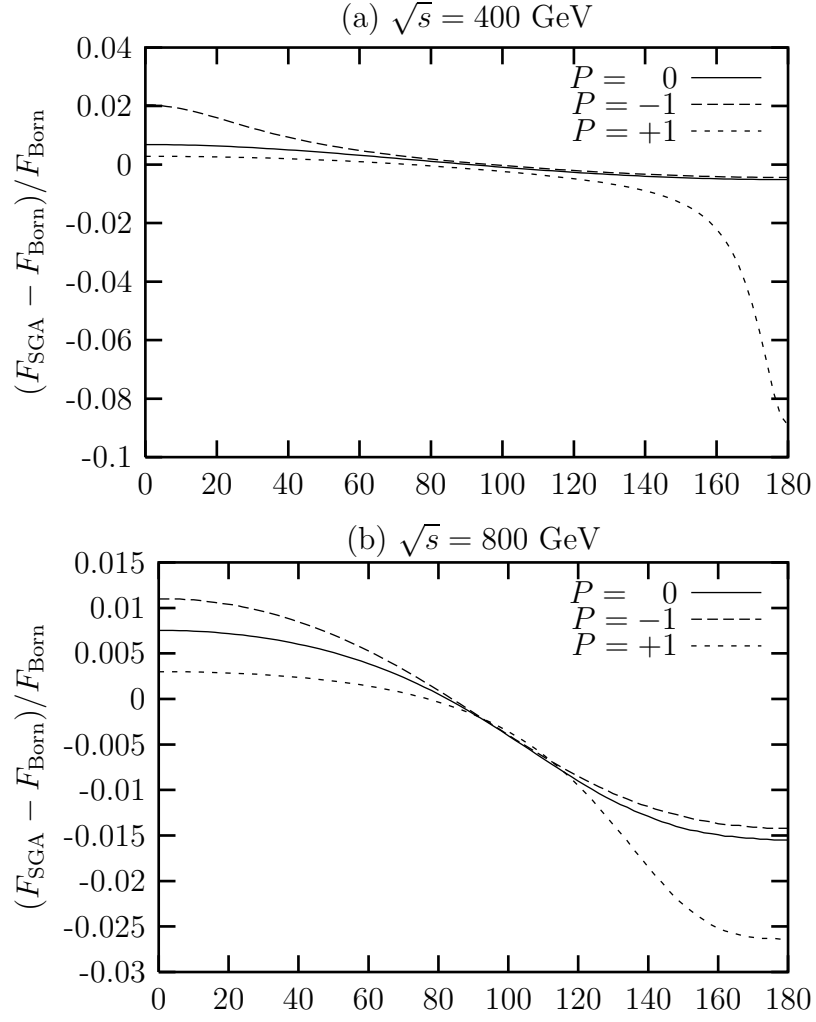
Figure 3: The fractional QCD contribution in normalized angular distributions, $F(\theta_l)$ defined in the text, for (a) $\sqrt{s} = 400$ GeV and (b) $\sqrt{s} = 800$ GeV plotted as a function of θ_l, for $P = 0, -1, +1$.

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polarized electron beam. For other values of $\sqrt{s}$, it is a slowly varying function of θ_l.

Though triple distributions in θ_t, θ_l and ϕ_l are not discussed in detail, it might be mentioned that they show an asymmetry about $\phi_l = 180°$, which is absent at tree level.

It would be useful to carry out the hard-gluon corrections explicitly and check if the soft-gluon approximation used here gives correct quantitative results.

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